Calcul des déformations des fils élastiques

Fils élastiques en arc de cercle - Accélération normale à l'axe du fil

Forces gravitationnelles agissant dans le plan du fil

Fil rond en cuivre

$d := 0.23 \cdot mm$ $\quad S := \pi \cdot \frac{d^2}{4}$ $\quad E := 1.1 \cdot 10^5 \cdot N \cdot mm^{-2}$ $\quad G := \frac{E}{2.6}$ $\quad \rho := 8.9 \cdot 10^3 \cdot kg \cdot m^{-3}$

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$J_t := J_{t\_circ}(d)$ $\quad I_{22} := I_{f\_circ}(d)$ $\quad I_{33} := I_{22}$

$W_t := W_{t\_circ}(d)$ $\quad W'_t := W_t$ $\quad W_{f2} := W_{f\_circ}(d)$ $\quad W_{f3} := W_{f2}$

Caractéristiques de l'arc de cercle $\quad R := 21.5 \cdot mm$ $\quad \psi_{AB} := 75 \cdot deg$

Forces extérieures en bout d'arc $\quad \psi_F := \psi_{AB}$ $\quad \psi_q := \psi_{AB}$

$F_x := 0 \cdot N$ $\quad F_y := 0 \cdot N$ $\quad F_z := 0 \cdot N$ $\quad C_x := 0 \cdot N \cdot mm$ $\quad C_y := 0 \cdot N \cdot mm$ $\quad C_z := 0 \cdot N \cdot mm$

Force distribuée gravitationnelle $\quad q_0 := \rho \cdot g \cdot S$ $\quad P_{fil} := q_0 \cdot R \cdot \psi_{AB}$ $\quad P_{fil} = 1.021 \times 10^{-4} N$

$\boxed{\lambda_g := 50 \cdot deg}$ $\quad q_x(\chi) := q_0 \cdot cos(\lambda_g)$ $\quad q_y(\chi) := q_0 \cdot sin(\lambda_g)$ $\quad q_z(\chi) := 0 \cdot N \cdot m^{-1}$

Référence :E:\Résonateur (TA)\Fils et lames en arc de cercle\Arc de cercle E_L - F&C&q.mcd(R)

Valeur de tests transitoires $\quad \alpha_m := 20 \cdot deg$

Torseur des forces de cohésion

$\mathbf{M_{cq}}(\psi_F, \psi_q, \alpha_m)^T = \begin{pmatrix} 0 & 0 & -7.37 \times 10^{-4} \end{pmatrix} N \cdot mm$

Sollicitations

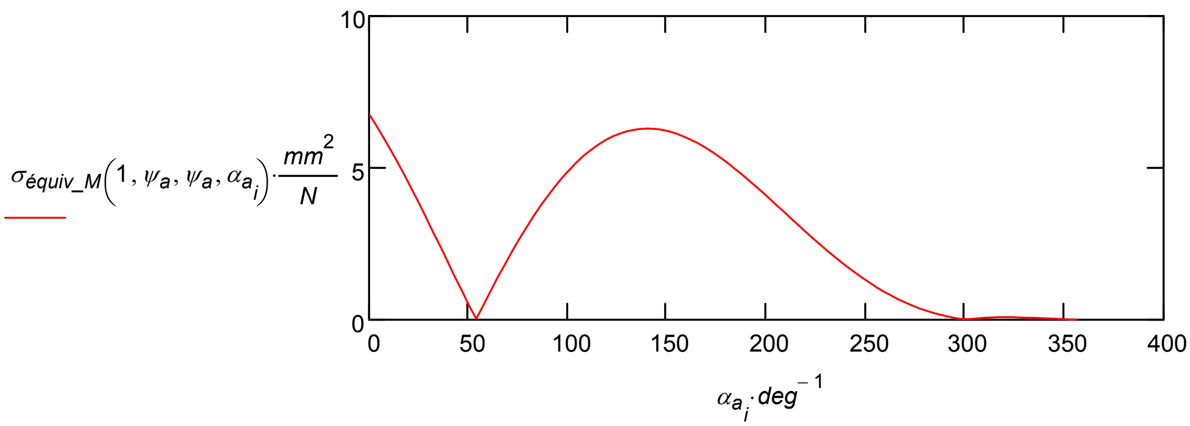
$\mathbf{e'_1}(\alpha_m)^T = (-0.342 \quad 0.94 \quad 0)$ $\quad \mathbf{e'_2}(\alpha_m)^T = (-0.94 \quad -0.342 \quad 0)$ $\quad \mathbf{e'_3}(\alpha_m)^T = (0 \quad 0 \quad 1)$

Moment de torsion $\quad M_t(\psi_F, \psi_q, \alpha_m) = 0\,N \cdot mm$

Moments de flexion $\quad M_{f2}(\psi_F, \psi_q, \alpha_m) = 0\,N \cdot mm$ $\quad M_{f3}(\psi_F, \psi_q, \alpha_m) = -7.37 \times 10^{-4} N \cdot mm$

Contraintes

Cas d'un anneau fendu $\quad n := 101$ $\quad i := 1 .. n-1$ $\quad \psi_a := 360 \cdot deg$ $\quad \alpha_{a_i} := (i-1) \cdot \frac{\psi_a}{n-1}$



Calcul des déplacements par les intégrales de Mohr

Position du déplacement désiré $\alpha_M := 40 \cdot deg$

Calcul des déplacements linéiques

Déplacement dans la direction de Ox $\lambda := 0 \cdot deg$ $\gamma := 90 \cdot deg$ $|\mathbf{v}(\lambda,\gamma)| = 1$

$\delta_{tv}(\psi_F,\psi_q,\alpha_M,\lambda,\gamma) = 0\,mm$ $\delta_{fv2}(\psi_F,\psi_q,\alpha_M,\lambda,\gamma) = 0\,mm$ $\delta_{fv3}(\psi_F,\psi_q,\alpha_M,\lambda,\gamma) = 6.02 \times 10^{-3}\,mm$

$\delta_x(\alpha) := \delta_v(\psi_F,\psi_q,\alpha,\lambda,\gamma)$ $\boxed{\delta_x(\alpha_M) = 6.02 \times 10^{-3}\,mm}$

Déplacement dans la direction de Oy $\lambda := 90 \cdot deg$ $\gamma := 90 \cdot deg$ $|\mathbf{v}(\lambda,\gamma)| = 1$

$\delta_{tv}(\psi_F,\psi_q,\alpha_M,\lambda,\gamma) = 0\,mm$ $\delta_{fv2}(\psi_F,\psi_q,\alpha_M,\lambda,\gamma) = 0\,mm$ $\delta_{fv3}(\psi_F,\psi_q,\alpha_M,\lambda,\gamma) = 2.87 \times 10^{-3}\,mm$

$\delta_y(\alpha) := \delta_v(\psi_F,\psi_q,\alpha,\lambda,\gamma)$ $\boxed{\delta_y(\alpha_M) = 2.87 \times 10^{-3}\,mm}$

Déplacement dans la direction de *R* $\lambda := \alpha_M$ $\gamma := 90 \cdot deg$ $|\mathbf{v}(\lambda,\gamma)| = 1$

$\delta_{tv}(\psi_F,\psi_q,\alpha_M,\lambda,\gamma) = 0\,mm$ $\delta_{fv2}(\psi_F,\psi_q,\alpha_M,\lambda,\gamma) = 0\,mm$ $\delta_{fv3}(\psi_F,\psi_q,\alpha_M,\lambda,\gamma) = 6.457 \times 10^{-3}\,mm$

$\delta_R(\alpha) := \delta_v(\psi_F,\psi_q,\alpha,\lambda,\gamma)$ $\boxed{\delta_R(\alpha_M) = 6.457 \times 10^{-3}\,mm}$

Calcul des déplacements angulaires

Déplacement angulaire autour de l'axe normal au plan de l'arc $\lambda_c := 0 \cdot deg$ $\gamma_c := 0 \cdot deg$ $|\mathbf{cv}(\lambda,\gamma)| = 1$

$\theta_{tcv}(\psi_F,\psi_q,\alpha_M,\lambda_c,\gamma_c) = 0\,deg$ $\theta_{fcv2}(\psi_F,\psi_q,\alpha_M,\lambda_c,\gamma_c) = 0\,deg$ $\theta_{fcv3}(\psi_F,\psi_q,\alpha_M,\lambda_c,\gamma_c) = -0.043\,deg$

$\theta_z(\alpha) := \theta_{fcv3}(\psi_F,\psi_q,\alpha,\lambda_c,\gamma_c)$ $\boxed{\theta_z(\alpha_M) = -0.043\,deg}$

Solution analytique

Moment fléchissant

$$M_{f3}(\psi,\alpha') := q_x(0) \cdot R^2 \cdot [\cos(\psi) - \cos(\alpha') + (\psi - \alpha') \cdot \sin(\alpha')] + q_y(0) \cdot R^2 \cdot [\sin(\psi) - \sin(\alpha') - (\psi - \alpha') \cdot \cos(\alpha')]$$

Déplacement selon Ox $M_v(\alpha,\alpha') := -R \cdot (\sin(\alpha) - \sin(\alpha'))$

$$\delta_{q1}(\psi,\alpha) := \frac{R}{E \cdot I_{33}} \cdot \int_0^{\alpha} M_{f3}(\psi,\alpha') \cdot M_v(\alpha,\alpha')\, d\alpha' \qquad \delta_{q1}(\psi_F,\alpha_M) = 6.02 \times 10^{-3}\,mm$$

$$I_{1x}(\psi,\alpha) := \left[\alpha \cdot \cos(\psi) + \psi - \frac{1}{2} \cdot (\psi - \alpha) \cdot \cos(\alpha) - \frac{5}{4} \cdot \sin(\alpha)\right] \cdot \sin(\alpha) - \frac{1}{2} \cdot \psi \cdot \alpha + \frac{1}{4} \cdot \alpha^2 - \cos(\psi) \cdot (1 - \cos(\alpha))$$

$$I_{2x}(\psi,\alpha) := \left[\alpha \cdot \sin(\psi) - 2 - \frac{1}{2} \cdot (\psi - \alpha) \cdot \sin(\alpha) + \frac{5}{4} \cdot \cos(\alpha)\right] \cdot \sin(\alpha) + \frac{3}{4} \cdot \alpha - \sin(\psi) \cdot (1 - \cos(\alpha))$$

$$\delta_{q1}(\psi,\alpha) := \frac{-R^4}{E \cdot I_{33}} \cdot \left(q_x(0) \cdot I_{1x}(\psi,\alpha) + q_y(0) \cdot I_{2x}(\psi,\alpha)\right)$$

Déplacement selon Oy $\qquad M_v(\alpha,\alpha') := -R\cdot(-\cos(\alpha)+\cos(\alpha'))$

$$\delta_{q2}(\psi,\alpha) := \frac{R}{E\cdot I_{33}}\cdot\int_0^{\alpha} M_{f3}(\psi,\alpha')\cdot M_v(\alpha,\alpha')\,d\alpha' \qquad \delta_{q2}(\psi_F,\alpha_M) = 2.87\times 10^{-3}\,mm$$

$$I_{1y}(\psi,\alpha) := \left[-\alpha\cdot\cos(\psi)+\frac{5}{4}\cdot\sin(\alpha)-\psi+\left(\psi-\frac{\alpha}{2}\right)\cdot\cos(\alpha)\right]\cdot\cos(\alpha)+\cos(\psi)\cdot\sin(\alpha)-\frac{3}{4}\cdot\alpha+\frac{1}{2}\cdot\psi\cdot\sin(\alpha)^2$$

$$I_{2y}(\psi,\alpha) := -\frac{3}{4}+\left[2-\alpha\cdot\sin(\psi)+(\psi-\alpha)\cdot\frac{\sin(\alpha)}{2}-\frac{5}{4}\cdot\cos(\alpha)\right]\cdot\cos(\alpha)+\frac{1}{4}\cdot\alpha^2+\sin(\psi)\cdot\sin(\alpha)-\frac{1}{2}\cdot\psi\cdot\alpha$$

$$\delta_{q2}(\psi,\alpha) := \frac{-R^4}{E\cdot I_{33}}\cdot\left(q_x(0)\cdot I_{1y}(\psi,\alpha)+q_y(0)\cdot I_{2y}(\psi,\alpha)\right)$$

Déplacement angulaire $\qquad M_v(\alpha,\alpha') := 1$

$$\theta_{q3}(\psi,\alpha) := \frac{R}{E\cdot I_{33}}\cdot\int_0^{\alpha} M_{f3}(\psi,\alpha')\cdot M_v(\alpha,\alpha')\,d\alpha' \qquad \theta_{q3}(\psi_F,\alpha_M) = -0.043\,deg$$

$$I_{1z}(\psi,\alpha) := \alpha\cdot(\cos(\psi)+\cos(\alpha))-2\cdot\sin(\alpha)+\psi\cdot(1-\cos(\alpha))$$

$$I_{2z}(\psi,\alpha) := \alpha\cdot(\sin(\psi)+\sin(\alpha))-2\cdot(1-\cos(\alpha))-\psi\cdot\sin(\alpha)$$

$$\theta_{q3}(\psi,\alpha) := \frac{R^3}{E\cdot I_{33}}\cdot\left(q_x(0)\cdot I_{1z}(\psi,\alpha)+q_y(0)\cdot I_{2z}(\psi,\alpha)\right)$$

Déplacements cartésiens totaux en M $\qquad \alpha_M = 40\,deg$

$$\boldsymbol{\Delta_q}(\psi,\alpha) := \begin{pmatrix}\delta_{q1}(\psi,\alpha)\cdot m^{-1}\\ \delta_{q2}(\psi,\alpha)\cdot m^{-1}\\ \theta_{q3}(\psi,\alpha)\end{pmatrix} \qquad \boldsymbol{\Delta_q}(\psi_F,\alpha_M) = \begin{pmatrix}6.02\times 10^{-3}\\ 2.87\times 10^{-3}\\ -0.745\end{pmatrix} 10^{-3}$$

Cas particuliers

Référence :E:\Résonateur (TA)\Fils et lames en arc de cercle\Définition Atan.mcd(R)

Quart de cercle

$\psi_{AB} := 90\cdot deg \qquad \lambda_g = 50\,deg \qquad L := R\cdot\psi_{AB} \qquad L = 33.772\,mm$

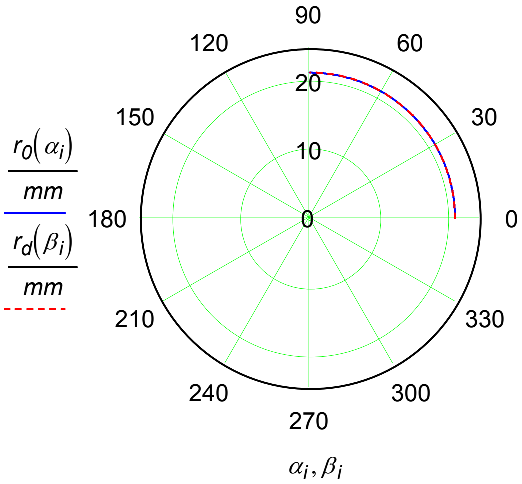
$\delta_{q1}(\psi_{AB},\psi_{AB}) = 0.02\,mm \qquad \delta_{q2}(\psi_{AB},\psi_{AB}) = 0.027\,mm \qquad \theta_{q3}(\psi_{AB},\psi_{AB}) = -0.083\,deg$

$$n := 201 \qquad i := 1..n \qquad \alpha_i := \frac{\psi_{AB}}{n-1}\cdot(i-1) \qquad x_0(\alpha) := R\cdot\cos(\alpha) \qquad y_0(\alpha) := R\cdot\sin(\alpha) \qquad r_0(\alpha) := \sqrt{x_0(\alpha)^2+y_0(\alpha)^2}$$

$$x_d(\alpha) := x_0(\alpha)+\delta_{q1}(\psi_{AB},\alpha) \qquad y_d(\alpha) := y_0(\alpha)+\delta_{q2}(\psi_{AB},\alpha) \qquad r_d(\alpha) := \sqrt{x_d(\alpha)^2+y_d(\alpha)^2}$$

$$\beta_i := Atan\left(x_d(\alpha_i),y_d(\alpha_i)\right) \qquad \beta_1 = 0\,deg \qquad \beta_n = 89.948\,deg$$

$$x'_d(\alpha) := \frac{d}{d\alpha}x_d(\alpha) \qquad y'_d(\alpha) := \frac{d}{d\alpha}y_d(\alpha) \qquad \alpha_0 := 0 \qquad \alpha_{max} := \psi_{AB}$$



$$L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2}\, d\alpha$$

$L_d = 33.772\, mm$ $\qquad$ $L = 33.772\, mm$

Cas d'un encastrement vertical (λ_g = 0°)

$$\Delta_{90V} := \frac{q_0 \cdot R^4}{E \cdot I_{33}} \cdot \left(\frac{5}{4} - \frac{\pi}{2} + \frac{\pi^2}{16} \quad \frac{\pi}{8} \quad \frac{\pi - 4}{2 \cdot R} \cdot m\right)^T \cdot \frac{1}{m} \qquad \Delta_{90V} = \begin{pmatrix} 0.015 \\ 0.02 \\ -1.024 \end{pmatrix} 10^{-3}$$

Cas d'un encastrement horizontal (λ_g = -90°)

$$\Delta_{90H} := \frac{q_0 \cdot R^4}{E \cdot I_{33}} \cdot \left(-3 + \frac{7 \cdot \pi}{8} \quad \frac{1}{4} - \frac{\pi^2}{16} \quad -\frac{\pi - 4}{2 \cdot R} \cdot m\right)^T \cdot \frac{1}{m} \qquad \Delta_{90H} = \begin{pmatrix} -0.013 \\ -0.019 \\ 1.024 \end{pmatrix} 10^{-3}$$

Demi-cercle

$\psi_{AB} := 180 \cdot deg$ $\qquad$ $\lambda_g = 50\, deg$ $\qquad$ $L := R \cdot \psi_{AB}$ $\qquad$ $L = 67.544\, mm$

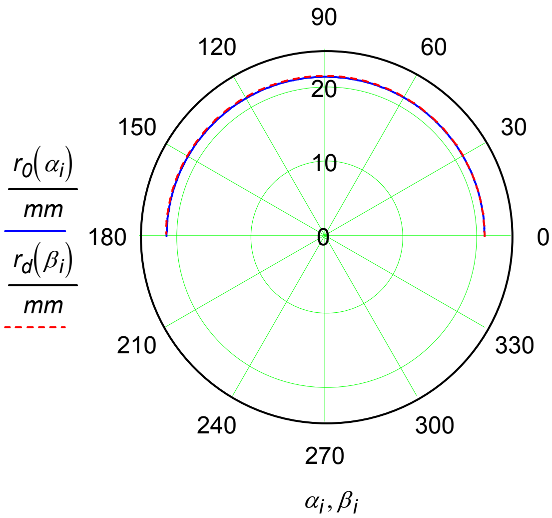
$\delta_{q1}(\psi_{AB}, \psi_{AB}) = -0.077\, mm$ $\qquad$ $\delta_{q2}(\psi_{AB}, \psi_{AB}) = 0.28\, mm$ $\qquad$ $\theta_{q3}(\psi_{AB}, \psi_{AB}) = -0.419\, deg$

Graphe de la déformation

$n := 201$ $\qquad$ $i := 1 .. n$ $\qquad$ $\alpha_i := \frac{\psi_{AB}}{n-1} \cdot (i - 1)$ $\qquad$ $x_0(\alpha) := R \cdot \cos(\alpha)$ $\qquad$ $y_0(\alpha) := R \cdot \sin(\alpha)$ $\qquad$ $r_0(\alpha) := \sqrt{x_0(\alpha)^2 + y_0(\alpha)^2}$

$x_d(\alpha) := x_0(\alpha) + \delta_{q1}(\psi_{AB}, \alpha)$ $\qquad$ $y_d(\alpha) := y_0(\alpha) + \delta_{q2}(\psi_{AB}, \alpha)$ $\qquad$ $r_d(\alpha) := \sqrt{x_d(\alpha)^2 + y_d(\alpha)^2}$

$\beta_i := Atan(x_d(\alpha_i), y_d(\alpha_i))$ $\qquad$ $\beta_1 = 0\, deg$ $\qquad$ $\beta_n = 179.257\, deg$



$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha)$ $\qquad$ $y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha)$

$\alpha_0 := 0$ $\qquad$ $\alpha_{max} := \psi_{AB}$

$$L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2}\, d\alpha$$

$L_d = 67.546\, mm$ $\qquad$ $L = 67.544\, mm$

Cas d'un encastrement vertical (λ_g = 0°)

$$\Delta_{180V} := \frac{q_0 \cdot R^4}{E \cdot I_{33}} \cdot \left(\frac{\pi^2}{4} - 2 \quad \frac{\pi}{4} \quad 0 \right)^T \cdot \frac{1}{m} \qquad \Delta_{180V} = \begin{pmatrix} 0.024 \\ 0.04 \\ 0 \end{pmatrix} 10^{-3}$$

Cas d'un encastrement horizontal (λ_g = -90°)

$$\Delta_{180H} := \frac{q_0 \cdot R^4}{E \cdot I_{33}} \cdot \left[\frac{3 \cdot \pi}{4} \quad -\left(4 + \frac{\pi^2}{4}\right) \quad \frac{4}{R} \cdot m \right]^T \cdot \frac{1}{m} \qquad \Delta_{180H} = \begin{pmatrix} 0.121 \\ -0.332 \\ 9.54 \end{pmatrix} 10^{-3}$$

Anneau fendu

$\psi_{AB} := 360 \cdot deg$ $\quad \lambda_g = 50\,deg$ $\quad L := R \cdot \psi_{AB}$ $\quad L = 135.088\,mm$

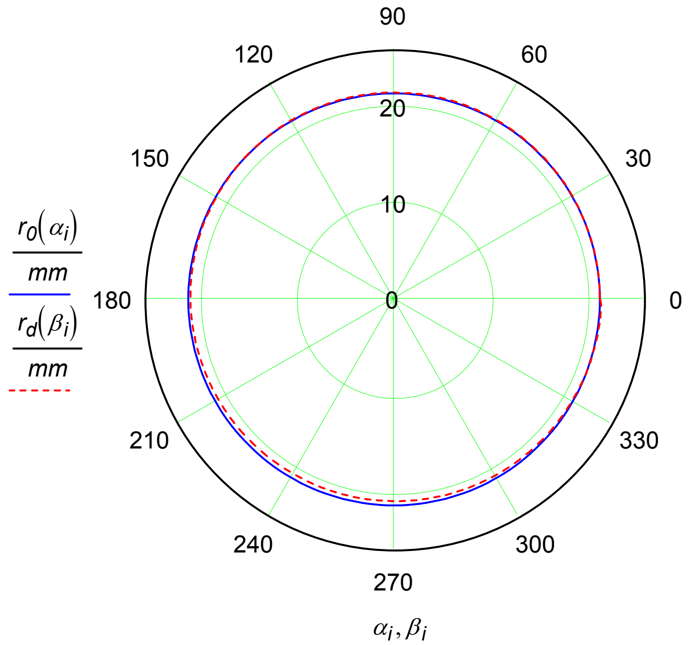
$\delta_{q1}(\psi_{AB}, \psi_{AB}) = 0.14\,mm$ $\quad \delta_{q2}(\psi_{AB}, \psi_{AB}) = 0.854\,mm$ $\quad \theta_{q3}(\psi_{AB}, \psi_{AB}) = 1.104\,deg$

Graphe de la déformation

$n := 201$ $\quad i := 1 .. n$ $\quad \alpha_i := \frac{\psi_{AB}}{n-1} \cdot (i-1)$ $\quad x_0(\alpha) := R \cdot cos(\alpha)$ $\quad y_0(\alpha) := R \cdot sin(\alpha)$ $\quad r_0(\alpha) := \sqrt{x_0(\alpha)^2 + y_0(\alpha)^2}$

$x_d(\alpha) := x_0(\alpha) + \delta_{q1}(\psi_{AB}, \alpha)$ $\quad y_d(\alpha) := y_0(\alpha) + \delta_{q2}(\psi_{AB}, \alpha)$ $\quad r_d(\alpha) := \sqrt{x_d(\alpha)^2 + y_d(\alpha)^2}$

$\beta_i := Atan(x_d(\alpha_i), y_d(\alpha_i))$ $\quad \beta_1 = 0\,deg$ $\quad \beta_n = 2.259\,deg$



$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha)$

$y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha)$

$\alpha_0 := 0$ $\quad \alpha_{max} := \psi_{AB}$

$$L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2}\, d\alpha$$

$L_d = 135.1\,mm$

$L = 135.088\,mm$

Cas d'un encastrement vertical (λ_g = 0°)

$$\Delta_{360V} := \frac{q_0 \cdot R^4}{E \cdot I_{33}} \cdot \left(\pi^2 \quad \frac{9 \cdot \pi}{2} \quad \frac{4 \cdot \pi}{R} \cdot m \right)^T \cdot \frac{1}{m} \qquad \Delta_{360V} = \begin{pmatrix} 0.506 \\ 0.725 \\ 29.971 \end{pmatrix} 10^{-3}$$

Cas d'un encastrement horizontal (λ_g = -90°)

$$\Delta_{360H} := \frac{q_0 \cdot R^4}{E \cdot I_{33}} \cdot \left(\frac{3 \cdot \pi}{2} \quad -\pi^2 \quad 0 \right)^T \cdot \frac{1}{m} \qquad \Delta_{360H} = \begin{pmatrix} 0.242 \\ -0.506 \\ 0 \end{pmatrix} 10^{-3}$$